

# An Appointment Overbooking Model To Improve Client Access and Provider Productivity

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# [ Appointment Scheduling ]



# [ Appointment Waiting ]



# [ Appointment Services ]

- Types of services
  - Medical care & mental health clinics
  - Dentists and medical specialists
  - Government offices; law offices
  - Counseling & admissions offices
  - Retail (tax help, salons, ...)
- We call these “*appointment services*”
  - Where “providers” in “offices” serve “clients”

# The “No-Show” Problem

- Research motivated by a outpatient mental health clinic in Denver, CO
  - 16 daily appointments / clinician
  - 30% no show rate
- Office no-show rates vary from 0-80%
  - <10% Brahim & Worthington (1991); Warden (1995)
  - 10-30% Barron (1980)
  - 3-80% Rust *et al.* (1995)

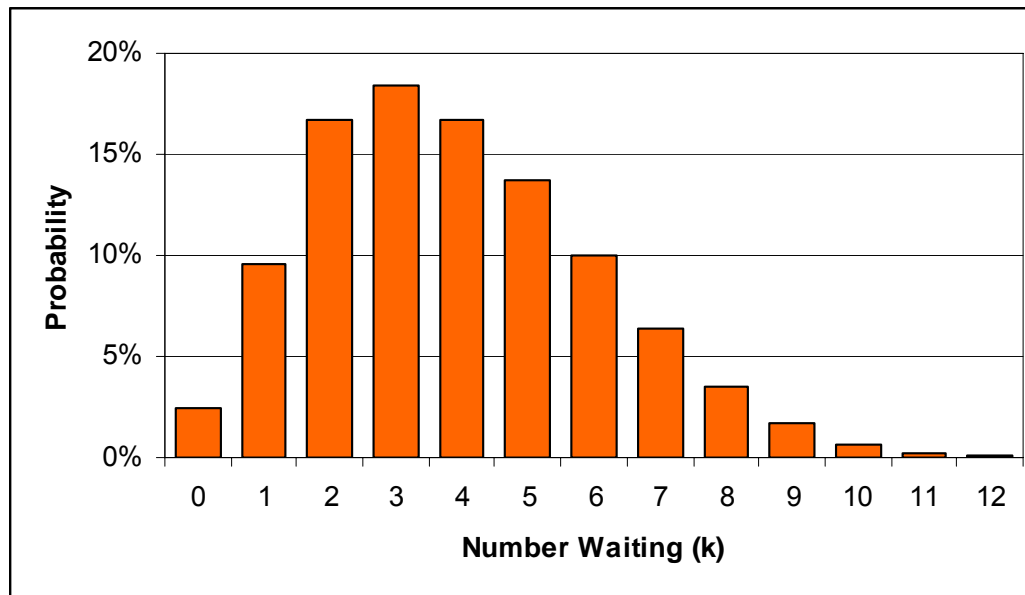
# Possible Solutions

- Sending clients reminder cards
  - Rust *et al* (1995)
- Call clients to remind them of appointments
- Providing public transport information
  - Bean & Talaga (1995)
- **Overbooking has not been closely examined as a possible response**
  - Widely used other businesses (*e.g.*, airlines)

# Literature

- Blanco White & Pike (1964)
  - Appointment systems in out-patients' clinics and effect of patients' unpunctuality. *Medical Care* 2(3), 133-145
- Vissers (1979)
  - Selecting a suitable appointment system in an outpatient setting. *Medical Care*, 17(12), 1207-1220
- Cayirli & Veral (2003)
  - Outpatient scheduling in health care: A review of the literature. *Production and Operations Management*, 12(4), 519-549
- LaGanga & Lawrence (2007a)
  - Clinic overbooking to improve patient access and increase provider productivity. *Decision Sciences*, 38(2).
- LaGanga & Lawrence (2007b)
  - "Appointment scheduling with overbooking to mitigate productivity loss from no-shows." *Proceedings of Decision Sciences Institute Annual Conference*, Phoenix, Arizona, November 17-20, 2007 (forthcoming)

# Appointment Scheduling and Overbooking Model



# How to Handle No-Shows?

- How to balance competing goals?
  - Provide better client access
  - Minimize client waiting
  - Minimize office overtime
  - Maximize provider productivity
- How to measure a “good” policy?
  - Is this a monetary problem?
  - A service problem?

# [ Overbooking Utility Model ]

- Maximize office “utility”
- Trade-off
  - Client access (number of clients seen)
  - Average client waiting times
  - Expected office overtime
- Note that provider productivity is implicit in this model

# [ Assumptions ]

- Clients “show” on time with probability  $\sigma$
- Client service times deterministic
  - No variability
- Clients serviced by assigned provider
- Office accrues
  - Benefits for serving additional clients
  - Penalties for keeping clients waiting
  - Penalties for office overtime

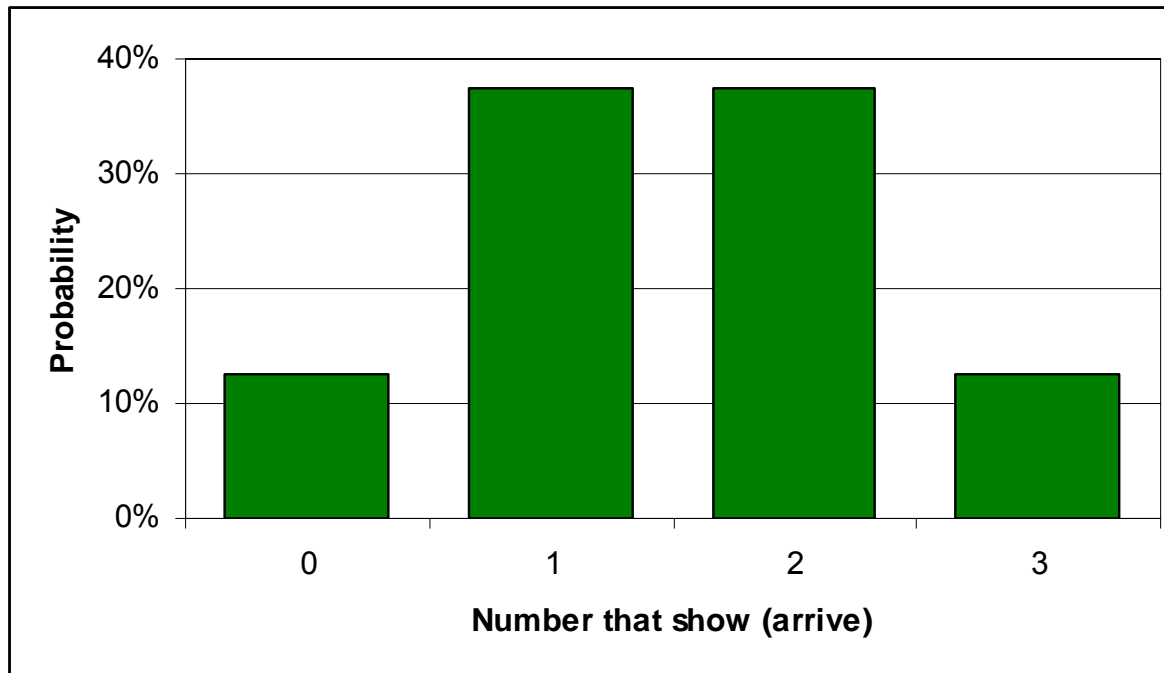
# Probability that $a_j$ Clients Arrive

- Arrivals are binomially distributed
  - $s_j$  clients scheduled for appt slot  $j$
  - Probability of a client showing is  $\sigma$
  - $a_j \leq s_j$  clients show for appointment

$$f(a_j; s_j, \sigma) = \binom{s_j}{a_j} \sigma^{a_j} (1-\sigma)^{s_j-a_j} = \frac{s_j!}{a_j!(s_j-a_j)!} \sigma^{a_j} (1-\sigma)^{s_j-a_j}$$

# Arrival Distribution Example

3 clients scheduled; 50% show rate



$N=162$ ,  $\sigma=50\%$ , slot  $j=12$ ,  $(\omega, \tau) = (0.5, 1.0)$  linear

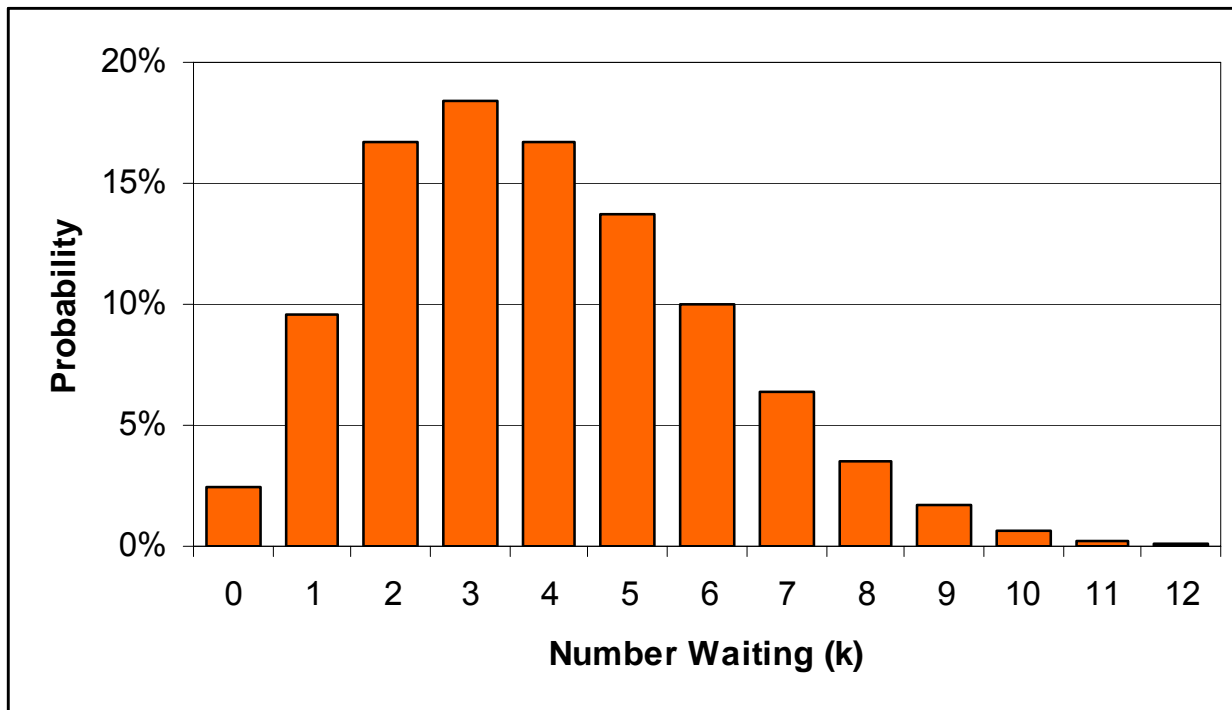
# [ Probability of $k$ clients Waiting ]

$$\theta_{j+1,k} = \theta_{j,0} \alpha_{j+1,k} + \sum_{i=0}^k \theta_{j,i+1} \alpha_{j+1,k-i}$$

- $\alpha_{jk}$  = probability of  $k$  clients arriving for service at the start of appointment slot  $j$
- $\theta_{jk}$  = probability of  $k$  clients waiting for service at start of appointment slot  $j$

# Number Waiting Example

Appointment slot 12; 3 clients scheduled

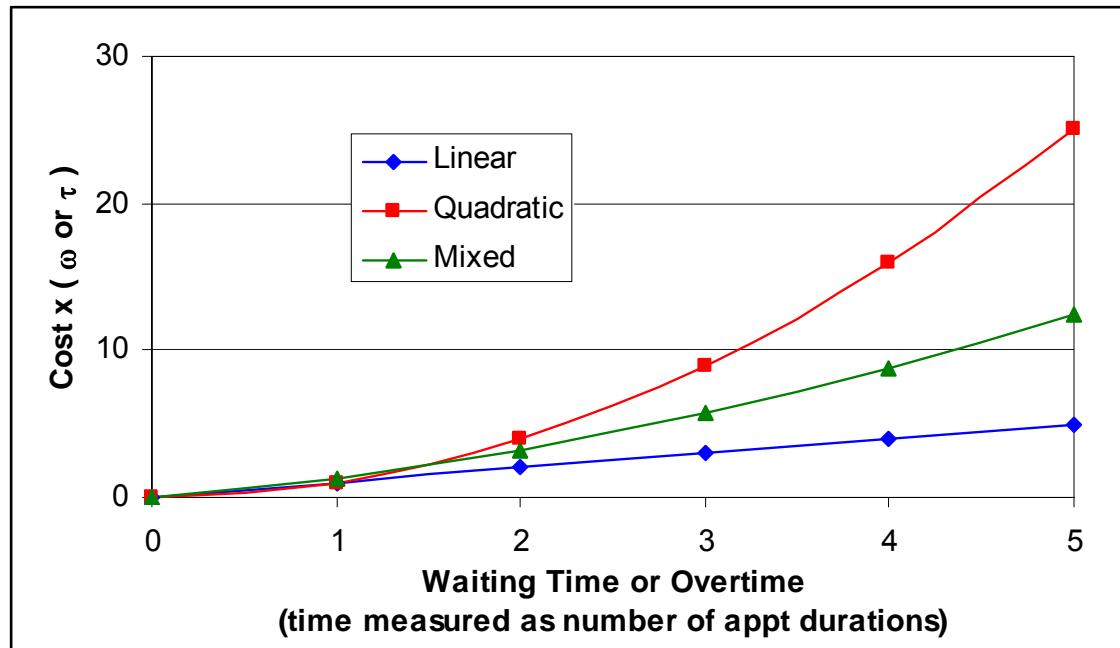


$N=16$ ,  $\sigma=50\%$ , slot  $j=12$ ,  $(\omega, \tau) = (0.5, 1.0)$  linear

# [ Relative Benefits and Penalties ]

- $\pi$  = Benefit of seeing additional client
- $\omega$  = Penalty for client waiting
- $\tau$  = Penalty for office overtime
- Numéraire of  $\pi$ ,  $\omega$ , and  $\tau$  doesn't matter
  - Ratios (relative importance) are important
- Allow both linear and quadratic costs

# Linear & Quadratic Costs



- Model allows 2<sup>nd</sup> order polynomials
  - Results not reported in this paper

# Linear & Quadratic Objectives

## Linear Utility Function

$$\hat{U}^L(\mathbf{S}) = \pi \hat{A} - \frac{\omega}{\hat{A}} \left( \sum_{j=1}^N \sum_k k \theta_{jk} + \sum_k \sum_{i=1}^k i \theta_{N+1,k} \right) - \tau \sum_k k \theta_{N+1,k}$$

## Quadratic Utility Function

$$\hat{U}^Q(\mathbf{S}) = \pi \hat{A} - \frac{\omega}{\hat{A}} \left( \sum_{j=1}^N \sum_k (2k-1) \theta_{jk} + \sum_k \sum_{i=1}^k i^2 \theta_{N+1,k} \right) - \tau \sum_k k^2 \theta_{N+1,k}$$

# [ Solution Methodology ]

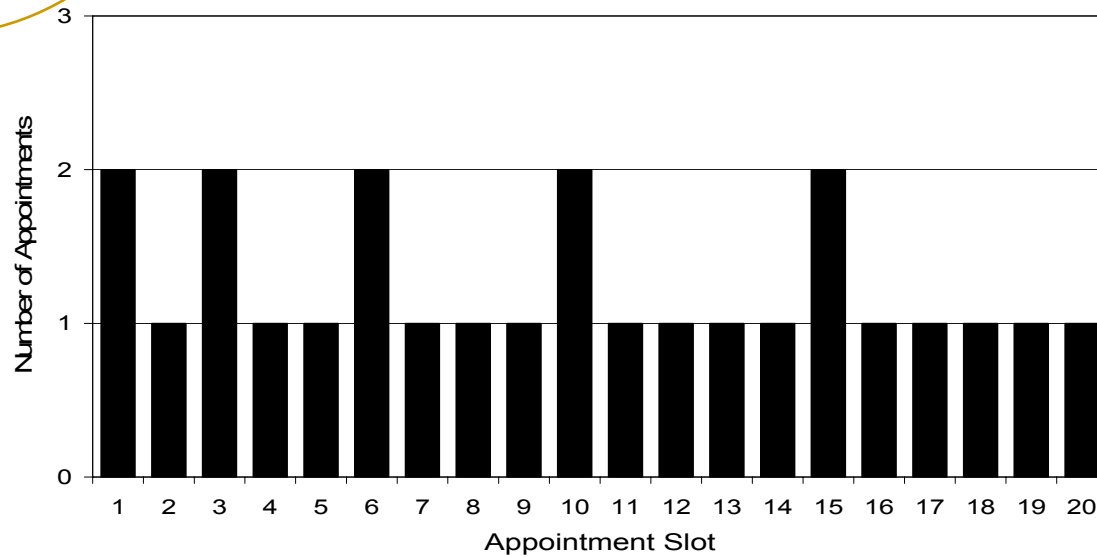
## 1. Gradient search

- Increment/decrement appointments scheduled in each slot
- Choose the single change which provides the greatest improvement in utility
- Iterate until no further improvement found

## 2. Pairwise interchange

- Exchange appointments scheduled in all appointment slot pairs
- Choose the single swap which provides the greatest improvement in utility
- Iterate until no further improvement found

# 3. Computational Results

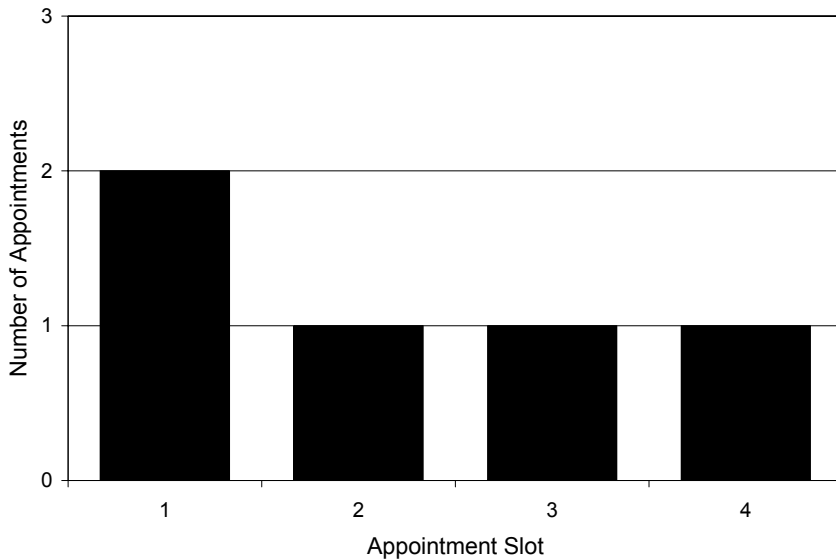


# Example Schedules

- 180 example problems solved
- Office sizes
  - $N = \{4, 8, 12, 16, 20, 24\}$
- Show rates
  - $\sigma = \{90\%, 80\%, \dots, 50\%\}$
- Benefit of additional client
  - $\pi = 1.0$
- Waiting / overtime costs
  - $(\omega, \tau) = (1.0, 1.0) (0.5, 1.5) (1.5, 1.5)$
- Linear and quadratic cost functions

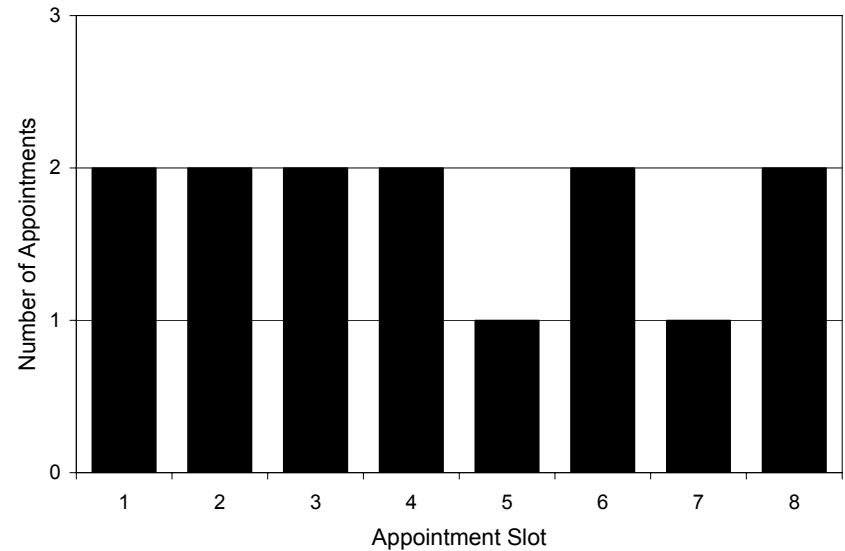
# [ Example Schedules (1/3) ]

**6A.**  $N=4$ ,  $\sigma=0.8$   
 $(\omega, \tau) = (0.5, 1.5)$  quadratic



- Front-loading
- Bailey (1952)

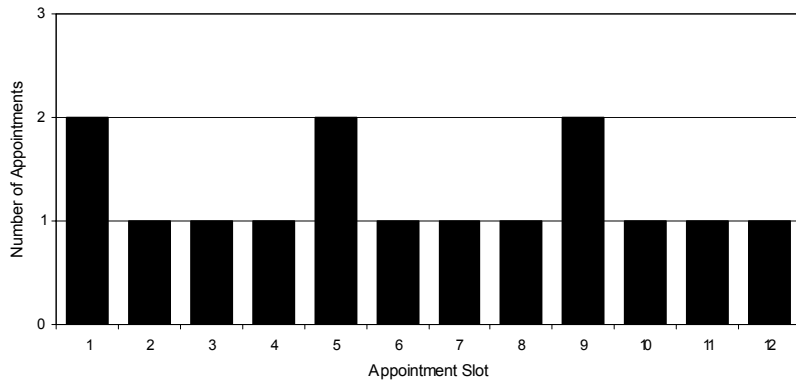
**6B.**  $N=8$ ,  $\sigma=0.5$   
 $(\omega, \tau) = (1.0, 1.0)$  linear



- Double-booking
- Welch & Bailey (1952)

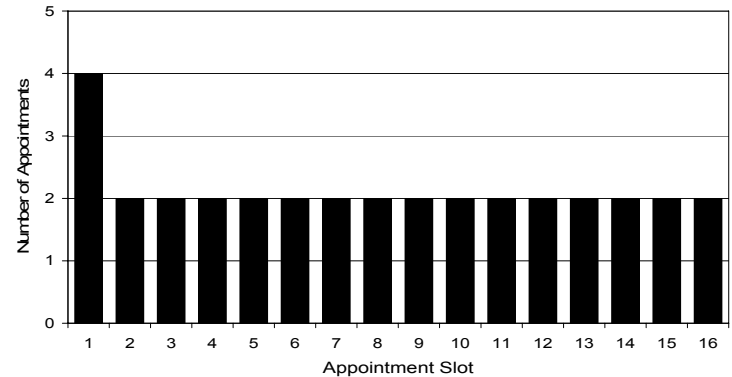
# Example Schedules (2/3)

**6C.**  $N=12$ ,  $\sigma=0.7$   
 $(\omega, \tau) = (1.0, 1.0)$  quadratic



- Wave schedule
- Baum (2001)

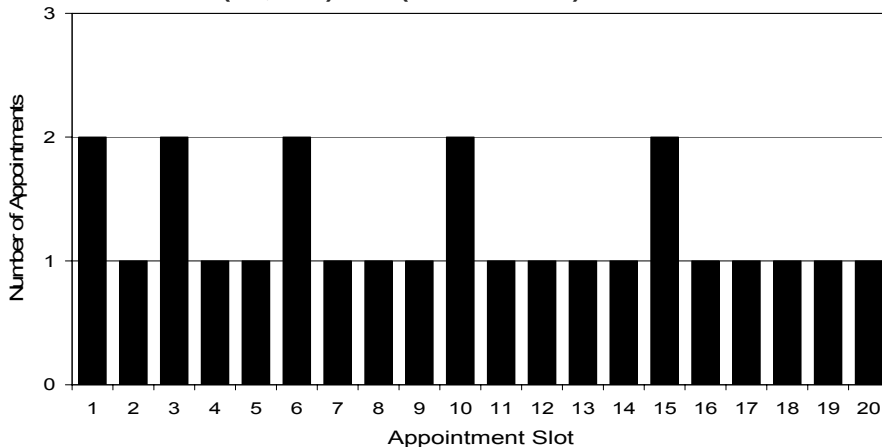
**6D.**  $N=16$ ,  $\sigma=0.5$   
 $(\omega, \tau) = (1.0, 1.0)$  linear



- Front-loading + double-booking

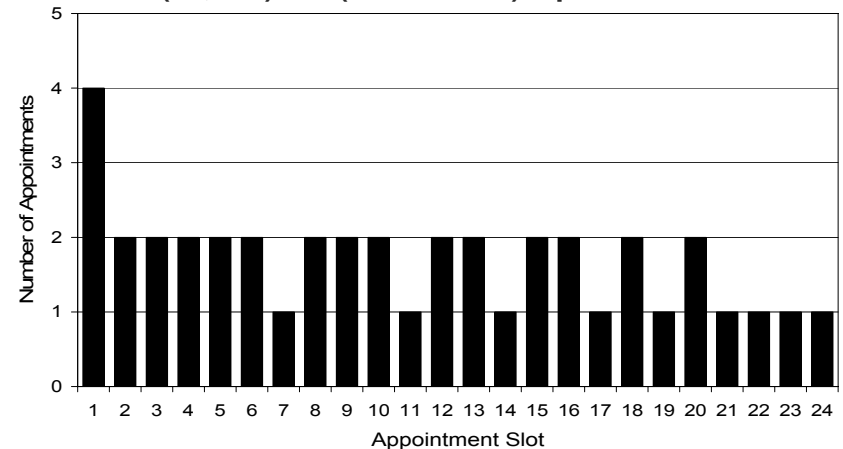
# [ Example Schedules (3/3) ]

**6E.**  $N=20$ ,  $\sigma=0.8$   
 $(\omega, \tau) = (0.5, 1.5)$  linear



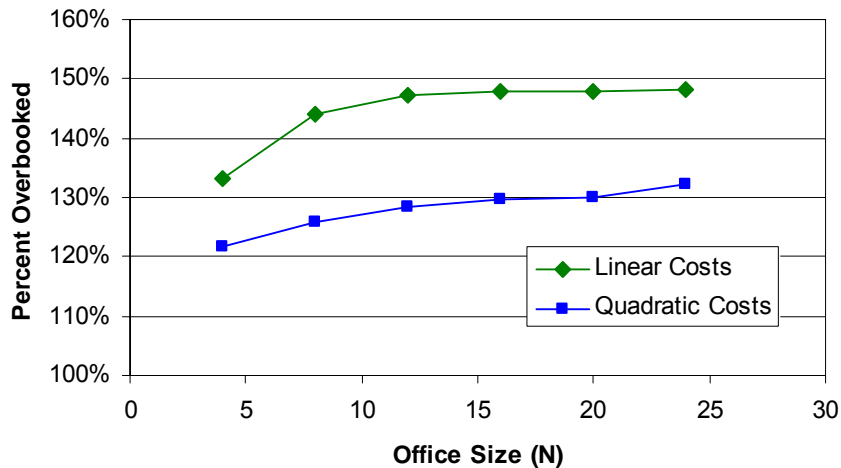
- Waves with increasing period

**6F.**  $N=24$ ,  $\sigma=0.5$   
 $(\omega, \tau) = (0.5, 1.5)$  quadratic

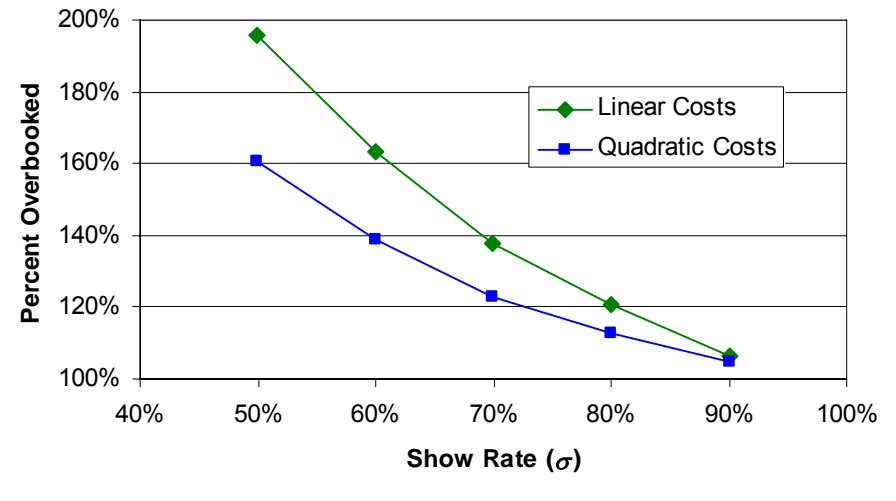


- Front-loading + double-booking + erratic waves

# Appointments Overbooked

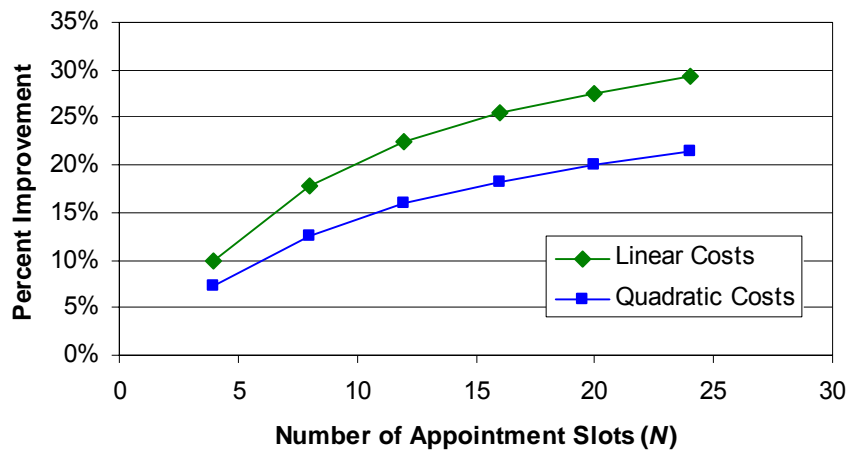


1A. Overbooking vs. office size  $N$

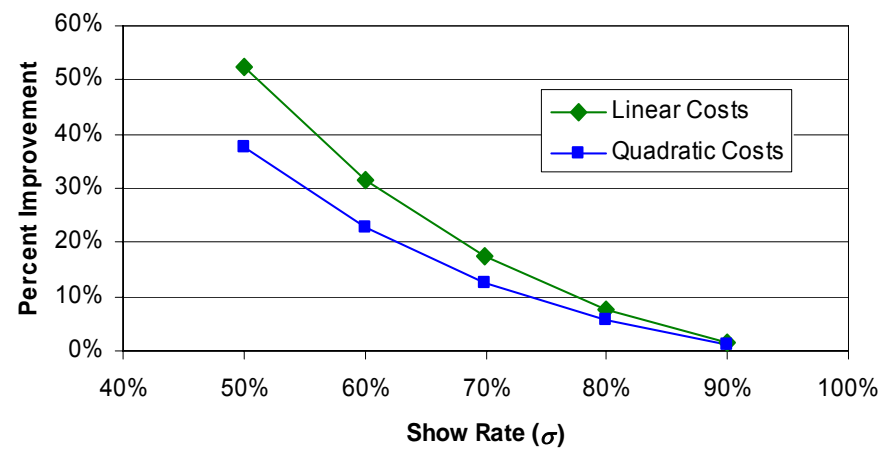


1B. Overbooking vs. show rate  $\sigma$

# Net Utility Improvement

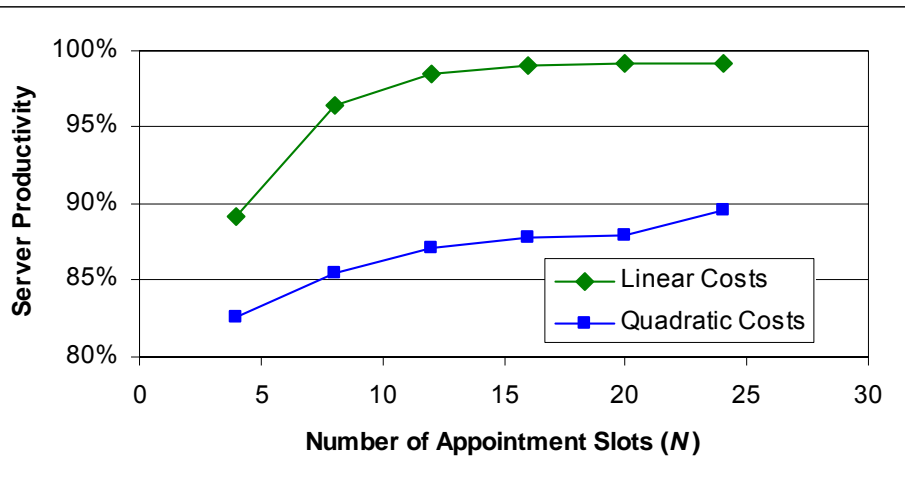


**2A.** Utility improvement vs. office size  $N$

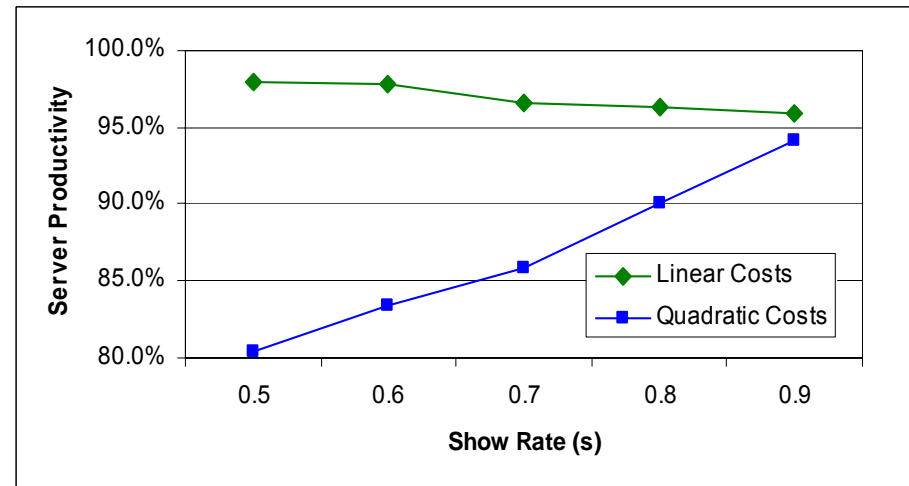


**2B.** Utility improvement vs. show rate  $\sigma$

# Server Productivity



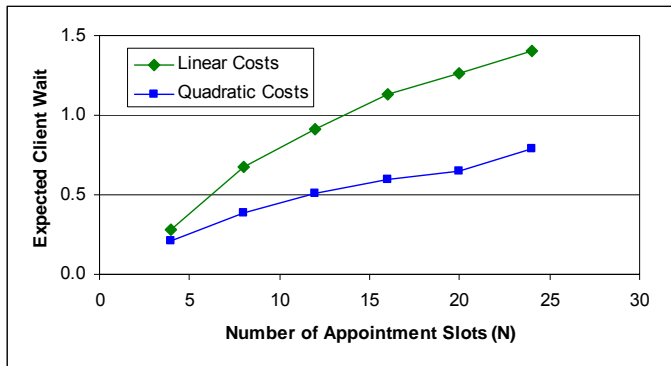
**3A.** Productivity vs. office size  $N$



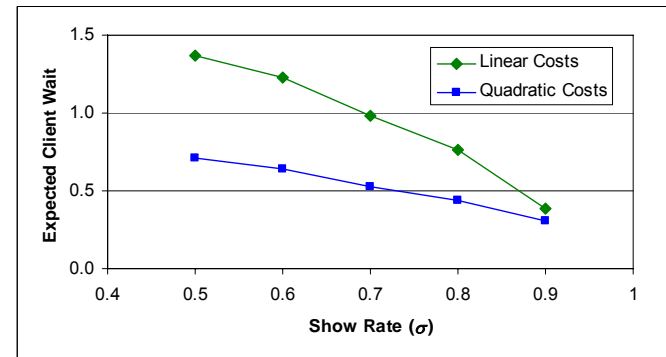
**3B.** Productivity vs. show rate  $\sigma$

Without overbooking, provider productivity is equal to the show rate  $\sigma$ .

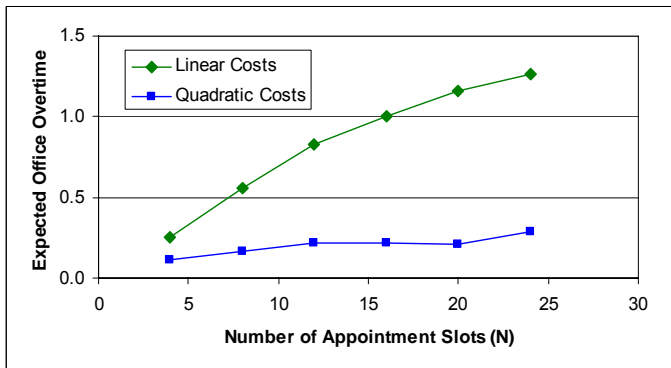
# Expected Waiting & Overtime



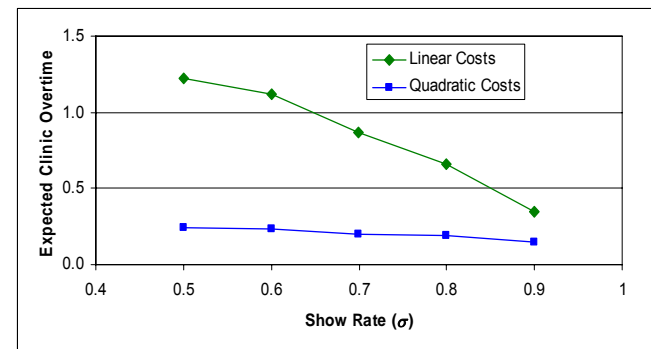
4A. Expected waiting vs. office size  $N$



4B. Expected waiting vs. show rate  $\sigma$

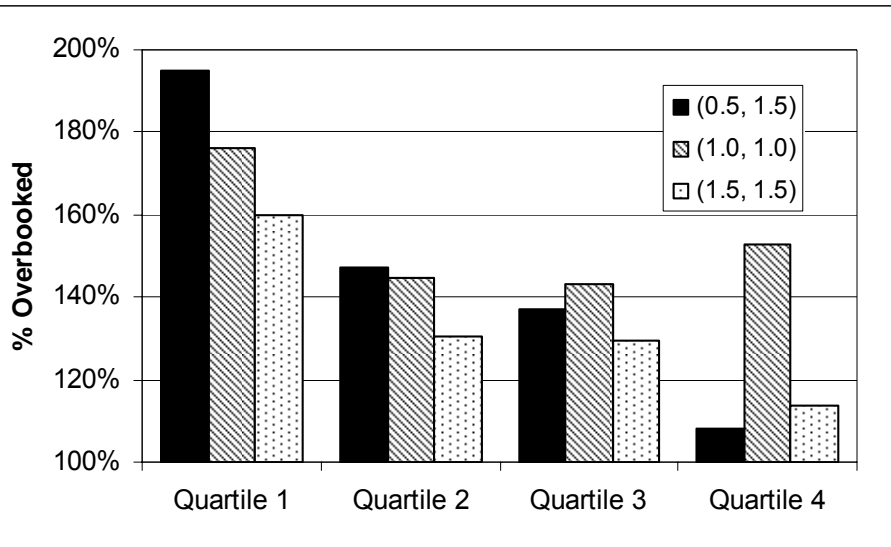


4C. Expected overtime vs. office size  $N$

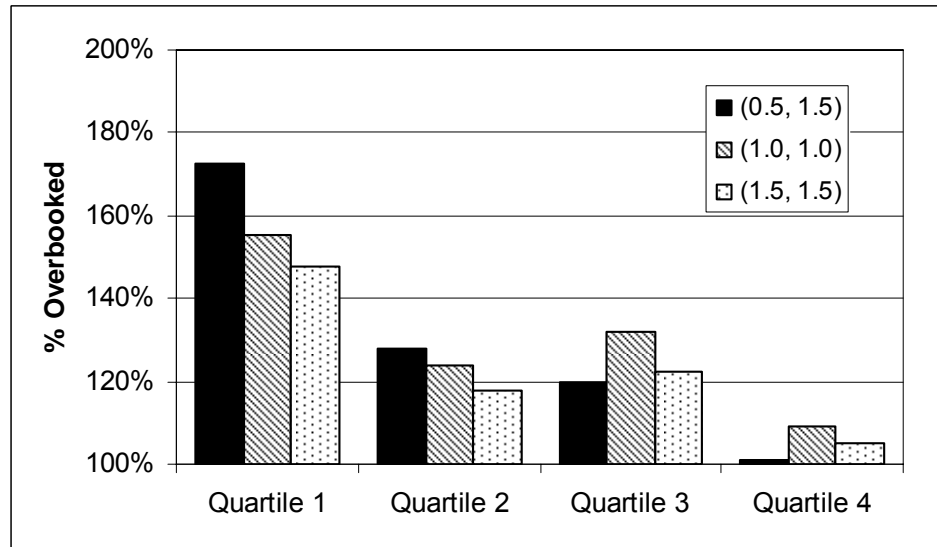


4D. Expected overtime vs. show rate  $\sigma$

# Overbooking Patterns



5A. Linear costs



5B. Quadratic costs

# [ Managerial Implications ]

- Overbooking (OB)
  - Improves customer service (serve more)
  - Increases provider utilization
  - Increases expected client wait times
  - Increases expected clinic overtime
- OB patterns are problem specific
  - Unlikely simple rules will satisfy
  - Need optimal or near-optimal schedules

# Contributions of Research

- Demonstrate benefits of appointment overbooking
- Analytic model of appointment scheduling with overbooking
  - Maximize utility
  - Balance service, waiting, and overtime
  - Linear and quadratic cost functions
- Fast and effective heuristic solutions
- Previous literature shown to be special cases of our analytic model

# [ Future Extensions ]

- Stochastic service times
- Service times vary by service type
- Show rates vary by time of day
- Appointments scheduled at any time
  - Not just at start of appointment slot
- Walk-ins

# Questions or Comments?

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